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Difficulties in the appropriation of the concept of computability in Systems Engineering students

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Abstract

The study was carried out in the city of Bogotá, during the first semester of the year 2020, at the higher education institution Escuela Tecnológica Instituto Técnico Central, in the subject *discrete mathematics*. The population consisted of 26 students. The mixed approach of the research has a sequential design and its objective was to analyze the relationship between the levels of appropriation of the concept of *computability* and the difficulties experienced by students around four variables. One of the main difficulties in the appropriation of the concept of computability comes from *the mathematics of computability* and specifically is related to the understanding of mathematical concepts, symbolism and algebraization; it also requires the holistic vision of the existing isomorphism between set theory, propositional logic, logical circuits and Boolean algebra. The results show that the difficulties characterized are close to the levels of appropriation (identified) around the concept of *computability*. The limitations corresponded to the crisis generated by the Covid 19, where it was not possible to apply the instruments in person. The teaching of mathematics in current times will not be the same as before, where the emphasis was on continuous mathematics; there is a new look towards the discrete and the finite. Finally, a series of guidelines for mathematics teaching in the XXI century in engineering faculties are proposed.

Keywords: Computability, discrete mathematics, Turing machine, didactics.

Introduction

Computers in the 21st century are the representation of a historical and social revolution, with mathematic sciences behind and *discrete mathematics* as its specialized branch, and computability as the cornerstone of this dynamic. This study approached *computability* beyond mathematics, its formalism and applicability, focusing on the teaching and learning processes with the objective of analyzing the relationship between the levels of appropriation of the concept of *computability* and the difficulties experienced by students in the discrete mathematics course of systems engineering at the Escuela Tecnológica Instituto Técnico Central - ETITC.

A look at both the teaching proposal and the learning process of mathematics was carried out, taking into account the lack of innovation that, according to Solar, García, Rojas and Coronado (2014), exists in higher education: "in university training courses it has been evidenced that the traditional didactic current is the protagonist, leading to a curriculum that is not very flexible in terms of innovation for the teaching of mathematics" (p.43).

Mathematics, due to its expansion during the last century, has undergone enormous transformations that require a new didactics, hence the emerging singularity of discrete mathematics insofar as it involves more than a training procedure; it implies the "understanding of the logical-deductive processes behind such calculations. Understanding the meaning of algorithm, computability, recursion and algorithmic computability among others" (Castro, 2011, p.89).

The results of this research contribute to the re-signification of the curriculum in the framework of the economic, scientific and technological dynamics worldwide, since it links high demand performance areas in the fourth technological revolution: software development, artificial intelligence, programming, automated processes and communication, among others (Medina & Moreno, 2016). The findings of this study enable the transformation of teaching practices based on the understanding of students' learning processes with respect to the appropriation of computability and its associated concepts (Ortiz & Simanca, 2016).

The concept of *computability* was approached from Hilbert's formalization project, with the tenth problem (1900) and the decision problem (1928), while the solution proposed by Alan Turing was taken as a reference. Logic was approached from the algebraic effort made by George Boole (1848), the theory of recursive functions from the lambda calculus elaborated by Alonso Church and Stephen Kleene (1936).

Methodology

The study was methodologically inscribed under the premises of mixed research. It sought to understand a phenomenon taking into account its complexity, from a shared and complementary point of view, addressing different levels of the problem. In this case, the difficulties associated with the appropriation of the concept of *computability* by students in the discrete mathematics course were described, based on two approaches: an objective one, guided by the quantitative method; and a subjective one, guided by the qualitative method.

The study was conducted in three phases (see Figure 1). The first one was developed by applying a test to a sample of 26 students, with the objective of identifying their levels (low, medium and high) of general and specific appropriation of the concept of computability. Regarding the second phase, a focus group was conducted with five students in order to establish the difficulties associated with the process of appropriation of the concept of computability. Finally, in the final phase, quantitative and qualitative results were correlated, seeking to generate a holistic understanding of the phenomenon studied.

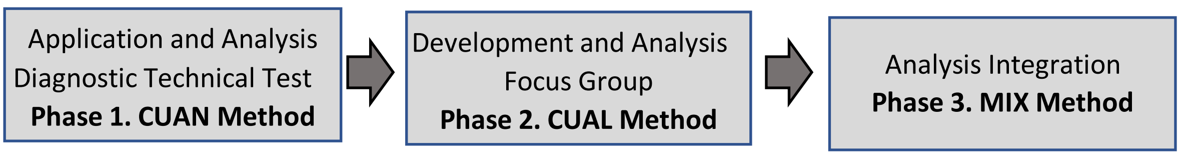

Figure 1. Phases of the methodology.

Source: authors' construction

The scope of the research was explanatory, since it specified the characteristics of the process of appropriation of the concept of *computability*, making visible its relationship with the factors that, according to the participants, affect the process; it sought to answer the following questions: what is the level of appropriation of the concept of *computability* in the students of the *discrete mathematics* course of the Systems Engineering program? and what are the factors that, according to the students, affect their learning of the concept of *computability*?

The participating population consisted of 26 students, 8 females and 18 males, whose ages range from 18 to 21 years old. They are students of the discrete mathematics course, which is taken in the second semester of the systems engineering career. Sample 1 is made up of 100% of the population, which corresponds to the 26 students of the subject Discrete Mathematics. Following the parameters of the quantitative method, the results were generalized with respect to a phenomenon (levels of appropriation of the concept of computability), through the application of a knowledge test. Sample 2 consisted of five students from the total population, who were part of the focus group (Table 1).

Table 1. *Description of the sample*

Sample 1					
Application of knowledge test (26 students)	100 % of the students of the subject "Discrete Mathematics".				
Sample 2	**Selection criteria**				
Focus group participation (5 students)	**Sex**		**Level of ownership**		
Participants	**M**	**F**	**Under**	**Medium**	**High**
Participant 1	x		x		
Participant 2		x	x		
Participant 3	x			x	
Participant 4		x		x	
Participant 5	x				x

Source: authors' creation

The instruments used in the first two phases were reviewed and approved by two experts. In phase one, a 38-question knowledge test was conducted, which was applied using Google Forms, on the following four variables that are specific to the concept of computability: Turing Machine, Recursive Functions, Algorithm and Computable Logic. The purpose of this initial phase was to establish the level of appropriation and its results were analyzed by means of descriptive statistics procedures. The instrument used in phase 2 was the questionnaire (14 questions), which was the basis for the development of the focus group through the Microsoft Teams application; it lasted 47 minutes and the data obtained were analyzed through axial coding and conceptual analysis mediated by the Atlas.Ti software.

Results and discussion

Figure 2 presents the overall percentages obtained in the knowledge test. The low level was established between 0 and 57% of assertiveness; the medium level between 58% and 79% of assertiveness; and the high level between 80% and 100% of assertiveness. If the 62% of the medium range with 19% of the high range is obtained, 81% of the population is reached with an adequate level of appropriation in the subject of *computability*, these results are in accordance with what is expected for second semester students that according to what is established in the syllabus of the institution (2020), "their competences must be based on doing from the skills and abilities, applying their integral formation in the basic sciences of engineering" (p.1) and more relevant in consideration that they are transversal subjects in systems engineering.

Figure 3 shows a general comparison of the four variables, being important to clarify that these topics are very different in terms of definition, understanding and application. This general comparison shows the dissimilarity of the results in the knowledge test with respect to the four variables that form a unit as far as computability is concerned.

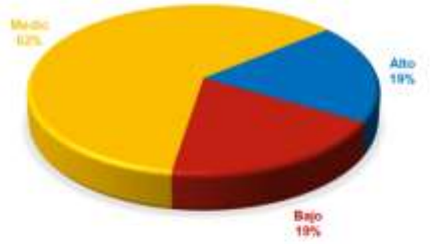

Figure 2. Global distribution scale by levels of ownership.

Source: *Own elaboration*

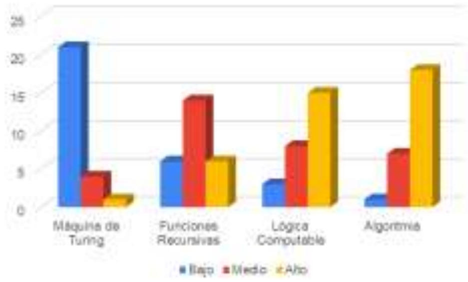

Figure 3. Global comparison of the four levels of ownership

Source: *Own elaboration*.

Mixed analysis

The following four matrices presented in Table 2, Table 3, Table 4 and Table 5 are the result of the mixed analysis, where a correlation between the quantitative method (vertical) and the qualitative method (horizontal) was performed. For each of the four variables, the respective sub-variables are presented in bold with white background. The difficulties are tabulated and classified in descending order from highest to lowest, with shades of gray; where the darkest gray corresponds to the highest level of difficulty and the lightest gray to the lowest level of difficulty.

Table 2. *Mixed Analysis Matrix - Variable Turing Machine.*

Variable	Subvariable/dificultades	Máquina de Turing – Análisis Cuantitativo: Niveles		
	Análisis Cualitativo	Bajo: 81%	Medio: 15%	Alto: 4%
Máquina de Turing	**Concepto matemático**			
	Comprensión del concepto (P3)	+	-	-
	Asociación entre programación y la máquina de Turing (P1)		+	-
	Desconocimiento del algoritmo (P2)	+	+/-	-
	Componentes			
	Identificación (P.4, P.5, P.6)	-	+	-
	Funcionamiento			
	Comprensión del funcionamiento (P.7, P.8)	+	+/-	-
	Desconocimiento del algoritmo y la lógica (P.9)	+/-	+	-
	Asociación entre el algoritmo y el funcionamiento de la máquina (P.10)	+	+/-	-

Source: Own elaboration.

Table 3. *Mixed Analysis Matrix - Recursive Functions Variable.*

Variable	Subvariables/dificultades	Funciones Recursivas – Análisis Cuantitativo: Niveles		
	Análisis Cualitativo	**Bajo: 23%**	**Medio: 54 %**	**Alto: 23 %**
Funciones recursivas	**Concepto de relación**			
	Comprensión del concepto matemático (P.11)	-	+/-	-
	Entendimiento de forma gráfica (P.12)	-	+	+/-
	Concepto de función			
	Identificación del concepto unicidad y existencia (P.13)	-	+/-	+
	Tipo de funciones			
	Identificación gráfica (P.15, P.16, P.17)	-	-	+
	Función computable			
	Asociación entre el proceso matemático y la fórmula (P.18)	+	-	-
	Concepto de función recursiva			
	Relación entre el concepto y sus aplicaciones (P.20, P.21)	+	-	-

Source: Own elaboration.

Table 4. *Mixed Analysis Matrix - Computable Logic Variable*

Variable	Subvariables/dificultades	Lógica computable – Análisis Cuantitativo: Niveles		
	A. Cualitativa	Bajo: 11%	Medio: 31%	Alto: 58%
Lógica computable	**Lógica proposicional**			
	Interpretación de símbolos lógicos (P.25)	+	+/-	-
	Relaciona con otras teorías matemáticas (P.22)	-	-	+
	Circuitos lógicos			
	Interpretación gráfica (P.26)	-	-	+
	Conjuntos			
	Comprensión de la identificación matemática (P. 30)	-	+	-
	Entendimiento de la forma gráfica (P.28)	-	+/-	+

Source: Own elaboration.

Table 5. *Mixed Analysis Matrix - Algorithmia Variable*

Variable	Subvariables/dificultades	Lógica computable – Análisis Cuantitativo: Niveles		
	Análisis Cualitativo	Bajo: 4%	Medio: 27%	Alto: 69%
Algoritmia	**Características de los algoritmos**			
	Comprensión asociada a la complejidad (P.32)	-	+/-	+
	Comprensión asociada a la lógica (P.31)	-	-	+
	Interpretación diagramas de flujo (P.33)	-	-	+
	Diagramas de flujo			
	Conocimiento conceptos básicos (P.35)	+/-	+	-
	Construcción de los algoritmos			
	Comprensión de la lógica y los símbolos asociados (P.36)	+	+/-	-

Source: Own elaboration.

The important part of each matrix is the associative results between the quantitative and qualitative analysis; their detail is found in the lower right part of each matrix, an area that is identifiable by the plus and minus signs in each of the four matrices.

The first variable Turing machine, its importance is emphasized since:

it is believed that the idea that the decision procedure is a mechanical procedure, i.e., it can be executed by an entity without intelligence, as long as it has access to the instructions indicated by it (which are created with intelligence) led Turing to conceive the idea of an abstract machine to describe this concept. This abstract machine is currently known as Turing's machine and corresponds to the notion of the concept of algorithm (Sicard, 1996, p.31).

According to the results, it had the lowest level of appropriation, related to its mathematical conceptualization, while the components had the highest assertiveness according to the quantitative results and the lowest difficulty with the qualitative results.

Regarding the variable recursive functions, it is important to note that this requires the highest level of mathematical formalization and today differs with respect to the concept of computability as mentioned by Mota:

Turing's computability is complemented by Alonzo Church's in terms of recursive functions and it is important to differentiate these two concepts because in the 1930s there was an attempt to associate the concept of computability with that of recursion and by the end of the twentieth century these concepts each had a life of their own and do not have equal meaning (2015, p,159).

On the other hand, with respect to formalization, an inference emerges more from the qualitative analysis when the students express that the graphic dimension makes the teaching of these subjects quite didactic.

In the third variable of computable logic, it has varied topics, therefore it is important what Freund (2011) expresses: “Conceptualism can play an important role in the resolution of problems of philosophy of mathematics or logic, it has also been employed as a foundation for the development of mathematical and logical theories” (p.10). In this context, the most relevant topic is the isomorphism of set theory, Boolean algebra, propositional logic and logical circuits. Graphical models (set theory) obtained the highest level of appropriation and the greatest difficulty with axiomatics, algebraization and symbolism.

The fourth variable *algorithm* presented the highest level of appropriation and understanding, due to the fact that the concept of algorithm in human culture is the oldest concept, reflected in the Rhind papyrus of four millennia ago and Euclid's algorithm of the classical Greek culture. The greatest difficulty is associated with the structuring of flowcharts, which agrees with the quantitative analysis expressed in the interview by the students:

Not all algorithms are easy to understand, an algorithm to add a number is not the same as an algorithm to identify a number or if a number is computable, each algorithm has its complexity and its study to understand it and depends a lot on this (Student interviewed).

Faced with the new knowledge required for the treatment of computability in *the knowledge society*, “the procedural thinking skills used in computer programming and linked to algorithmic thinking appear as an opportunity for the development of problem-solving skills” (Barrera, 2013).

Conclusions

Computability, understood from systems engineering, is a structuring and transversal concept both in curricular training and in the applications of the multiple branches of technology, which was made explicit from mathematics in the 30s of the last centuries from the work of pioneers such as Turing, Church, Post, Gödel, Kleene and Markov, among others. Although computability is specific to systems engineering, it is used in all engineering and technical fields, including art.

From a quantitative point of view, the levels of ownership generally averaged 62%, regardless of gender.

With respect to the four variables established in the study, the Turing machine was the determinant for having a low level in the overall score. At medium level are the two variables recursive functions and computable logic in their order; and at high level, the algorithm variable was the one that best contributed to the overall score.

The main difficulties in the appropriation of the concept of computability come from the mathematics of computability, and specifically have to do with the understanding of mathematical concepts, axiomatization, symbolism and algebraization.

In general, the results show that the difficulties characterized are close to the levels of appropriation (identified) around the concept of computability.

Teaching Turing's model to first semester students is not only important for the very definition of computability, but also allows to recover Alan Turing's synthetic vision, who reduces the complexity of computability to an operational mechanical model. This is important because the subject appears in the last semesters of most curricula.

A didactic sequence is proposed for the teaching of discrete mathematics structured in the following topics: (a) Set theory, (b) Computable sets, (c) Logic gates, (d) Turing machine, (e) Propositional logic, (f) Computable logic, (g) Recursive functions, (h) Computable functions, (i) Algorithmics. Items d.) and g.) are the topics that should be studied in depth, with more teaching hours in theory and practice, since it was in these topics where the lowest levels of appropriation were presented, not only from discrete mathematics but also from the other subjects.

The imprint of the technological revolution in the 21st century requires deconstructing the black box of devices, the scientific-technical training of young students who opt for technology as their professional training requires this effort and the country needs it in order not to be mere consumers in a globalized world.

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